Drakkenheim Contamination Mortality

Assume that a player leaves the Drakkenheim haze with $c \in [0, 9]$ contamination levels and $e \in [0, 5]$ exhaustion levels. They further have to wait $r \in \mathbb{N}_0^+$ long rests until they can use the *Purge Contamination* spell. We want to calculate their chance of dying as a direct result of contamination, assuming that they can use the *Purge Contamination* spell as necessary after r long rests.

Rule Recap

- Exhaustion is reduced by 1 after a long rest.
- If $c \leq 3$, contamination is reduced by 1 after a long rest.
- If $c \in \{4, 5\}$, contamination is reduced by 1 after 7 long rests.
- If $c \geq 6$, there is a $\frac{c}{20}$ chance to gain 1 contamination level after a long rest.
- Use of the *Purge Contamination* spell sets new $c' = 0$ and $e'(c, e) = e + \lceil \frac{c}{2} \rceil$.

Solution

Let $p(c, e, r)$ denote the chance of death at c contamination levels and e exhaustion levels, with the *Purge Contamination* spell being available after r long rests.

By the rules, if $c > 9$, then $p(c, e, r) = 1$. Also, if $c \leq 5$, then $p(c, e, r) = 0$.

For now, assume that $r = 0$, i.e. the player can immediately purge contamination. If $e = 0$, then all contamination can be purged, so $p(c, 0, 0) = 0$.

Now let $c \geq 6$ and assume the player has too much exhaustion to accommodate for potentially purged contamination. They will have to wait at least one long rest, after which the player loses one level of exhaustion. Additionally, the player has a $\frac{c}{20}$ chance of gaining one contamination level, leaving them at a $p(c + 1, \max(e - 1, 0), 0)$ chance of death. Otherwise, they are left at a $p(c, \max(e - 1, 0), 0)$ chance of death. In total, we get

$$p(c, e, 0) = \frac{c}{20} p\Big(c + 1, \max(e - 1, 0), 0\Big) + \frac{20 - c}{20} p\Big(c, \max(e - 1, 0), 0\Big).$$

$p(c, e, 0) = 0$ also holds simply if e is low enough to accommodate for any purged contamination, i.e. if $e'(c, e) \leq 5$. Note that this is already covered since in these cases, the player could also wait until no exhaustion is left and then purge all contamination.

We still have to cover the case that $r \neq 0$. If the player has to wait one long rest, they loose one exhaustion level and again have a $\frac{c}{20}$ chance of gaining one contamination level. Afterwards, they still have $r - 1$ long rests left. In total, we get

$$p(c, e, r) = \frac{c}{20} p\Big(c + 1, \max(e - 1, 0), r - 1\Big) + \frac{20 - c}{20} p\Big(c, \max(e - 1, 0), r - 1\Big).$$

We visualize the chance of death $p(c, e, r)$ in percent in the following tables, assuming that the player leaves the haze with c contamination levels, e exhaustion levels, and has to wait r long rests for the *Purge Contamination* spell. *P* indicates that contamination can be purged immediately and there is thus no chance of death.

$r = 0$

c \ e	0	1	2	3	4	5
0	P	P	P	P	P	P
1	P	P	P	P	P	0
2	P	P	P	P	P	0
3	P	P	P	P	0	0
4	P	P	P	P	0	0
5	P	P	P	0	0	0
6	P	P	P	0	2	7
7	P	P	0	6	18	31
8	P	P	18	39	57	70
9	P	45	70	83	91	95

$r = 1$

c \ e	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	2	7
7	0	0	0	6	18	31
8	0	0	18	39	57	70
9	45	45	70	83	91	95

$r = 2$

c \ e	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	2	7
7	0	0	0	6	18	31
8	18	18	18	39	57	70
9	70	70	70	83	91	95

$r = 3$

c \ e	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	2	7
7	6	6	6	6	18	31
8	39	39	39	39	57	70
9	83	83	83	83	91	95

$r = 4$

c \ e	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	2	2	2	2	2	7
7	18	18	18	18	18	31
8	57	57	57	57	57	70
9	91	91	91	91	91	95

$r = 5$

c \ e	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	7	7	7	7	7	7
7	31	31	31	31	31	31
8	70	70	70	70	70	70
9	95	95	95	95	95	95

$r = 6$

c \ e	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	14	14	14	14	14	14
7	45	45	45	45	45	45
8	80	80	80	80	80	80
9	97	97	97	97	97	97

$r = 7$

c \ e	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	23	23	23	23	23	23
7	57	57	57	57	57	57
8	87	87	87	87	87	87
9	98	98	98	98	98	98

$r = 8$

c \ e	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	33	33	33	33	33	33
7	68	68	68	68	68	68
8	92	92	92	92	92	92
9	99	99	99	99	99	99